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A STUDY OF INITIAL AND BOUNDARY VALUE PROBLEMS OF DIFFERENTIAL INCLUSIONS INVOLVING DISCONTINUITIES

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Origin Of The Research Problem

The problems in general areas of physical and natural explain to the study of dynamical systems having velocities not uniquely determined by the state but loosely upon it. In these circumstances, the mathematical model describing the dynamical system in question may be replaced by an inclusions of the form ,where is a multi-valued mapping on the suitable domain of the definition. The initial motivation for the differential inclusions comes from control theory, the dynamical systems described by the equations of the form being the control parameter. Every solution of this equation , also solves the differential inclusion,where . In mathematics, differential inclusions are a generalization of the concept of ordinary differential equation of the form

$\frac{dx}{dt}(t) \in F(t, x(t)),$ where F is a multivalued map.Differential inclusions arise in many situations including differentialvariational inequalities ,projected dynamical systems, dynamic Coulomb friction problems and fuzzy set arithmetic.For example, the basic rule for Coulomb friction is that the friction force has magnitude μN in the direction opposite to the direction of slip, where N is the normal force and μ is a constant (the friction coefficient). However, if the slip is zero, the friction force can be *any* force in the correct plane with magnitude smaller than or equal to μN. Thus, writing the friction force as a function of position and velocity leads to a set-valued function.

It appears that the control system and differential inclusion have the same trajectories of the sets , control depends on that is then we obtain the differential

inclusion. The equivalence between the control system and corresponding differential inclusion is the central idea used to prove the existence theorem in optimal control theory. Since the dynamics of economical,social and biological macrosystems are multi-valued differential inclusions, serve as natural models in macro system dynamics. Differential inclusion is a generalization of the notion of an ordinary differential equation. Therefore all problems considered for differential equations that is existence of solutions, continuation of solutions, dependence on initial conditions or boundary conditions and parameters are present in the theory of differential inclusions. The differential inclusions usually has many solutions starting at a given point, new issues appear, such as investigation of topological properties of the set solutions, selection of solution with given properties, evaluation of the reachability sets etc. and to solve the above problems special mathematical techniques were developed. Thus, the differential inclusions are not only models for many dynamical processes but they also provide a powerful tool for various branches of mathematical analysis.

Review of Literature

The theory of differential inclusions was initiated in 1934–1936 by the French mathematician A. Marchaud and S. K. Zaremba a mathematician from Cracow.The rapid development of the theoryat the beginning of the sixties of the century, when Cracow Mathematical Schoolby TadeuszWazewski[23] started working on these issues.

The following mathematicians belonging to the group are worth mentioning Andrzej Lasota and Zdzisław Opial [21], Czesław Olechand and Andrzej Plis [22] ,Józef Myjak [8], Andrzej Pelczar [24].The theory of differential inclusions is located within the mainstream of non-linear analysis or to put it more precisely

multi-valued analysis. This theory is intensively developed especially in the countries such as France, Germany, Russia, Italy, Canada ,Poland and USA.

The differential inclusions appear quite naturally in the study of diffusions process and Brownian motion. It is known that many of the natural and physical phenomena of the universe are non-linear in nature and involve the discontinuities or jumps with respect to the time variable. Therefore it is of interest to study the non-linear differential inclusions or inclusion and their application s to day-to-day problems of the life. The linear as well as non-linear differential inclusions or inclusions have been studied extensively in the literature by the several authors for various aspects of the solutions by long time. Several researchers have studied on this by direction via classical fixed point theory, Approximation method and Measurable selection method. Many researchers working in this direction and written a research papers and monographs on this topic by J. Aubin and Cellina [2] and many authors in research papers F. B. De Blassi and J. Myjak [8], C. Bonnano and S.A. Morrano [9], M. Benalim,J. Hofbayer and S. Sorin [5], H.J. Kushner and G.G. Yin [18], L. Ljung[20], M. Benchora and S.K.Ntouyas [4].The topic of differential equations is quite old as that of Famous Mathematician Isac Newton. It is much developed subject in both pure as well as applied branches of mathematics. In the beginning, the subject of differential inclusions was considered to be a part of applied mathematics, however in the latest stage, in has become most pure part of the pure mathematics. Also, the differential inclusions can be used to understand and suitably interpret discontinuous ordinary differential equations, such as arise for Coulomb friction in mechanical systems and ideal switches in power electronics. An important contribution has been made by A. F. Filippov [17], who studied regularizations of discontinuous equations. Further the technique of regularization was used by N.N. Krasovskii [19] in the theory of differential games.

OBJECTIVES

The present topic highlights the following valuable achievements,

1. To investigate the existence results of initial and boundary value problems for ordinary differential inclusions.
2. To investigate the existence results of initial and boundary value problems for ordinary integro-differential inclusions.
3. Qualitative study of solutions of differential inclusions.
4. Applications.

Methodology

There are mainly two approaches for dealing with the non-linear ordinary differential inclusions via,

1. Measurable Selection Method, and
2. Fixed point Method.

In the first approach of measurable selection method is very interesting, but it has a limitation that it is necessary to known in advance that the differential inclusions in question has a deterministic solution. Then, the measurability of this solution with respect to the parameter is proved by applications of some known results from probabilistic functional analysis. E Papageorgiou in, On the measurable selection approach in differential inclusions, fixed point theory and optimization.

The second approached of fixed point method is the most powerful one which on application directly yields the existence of solutions. There are many fixed point theorems are in existence as Schauder Fixed Point Theorem, Schafer Fixed Point Theorem ,Browder Potter Fixed Point Theorem, Kransoselskii's Fixed Point Theorem, Peano Fixed Point Theorem, Kakutani's Fixed Point Theorem, Leray-Schauder Theorem. The work in this direction may be foundin J. Aubin and ACellina ,S. K. Ntouyas ,N.S.Papagergiou,D.O'Regan, B.C.Dhage,Bharucha- Reid in Random Integral Equations.

IMPORTANCE OF STUDY

Significance of the study

The present work is new to the literature and results of this work will be a good contribution to the study of differential inclusions.

Since the study of differential inclusions will be useful to Scientist, Engineers and Mathematicians etc., who are engaged in analyzing universal phenomena covered by the nonlinear initial or boundary value problems of ordinary differential inclusions. As a research field, differential inclusions may be regarded as gymnasium,where new ideas and techniques can be introduced and developed. These ideas have found application to other branches of mathematics as control theory, calculus of variations, PDEs.

Status: International

Differential inclusions and study of its solution is completely new topic in the research area. Mathematical modeling of differential inclusion is studied in an international level long time by Einstein in 1905, Smolouchowski in 1906,Langevin in 1908,Ornstein and Uhlenbeck in 1930 and till recently many

researcher work on differential inclusions of all fields in natural and physical sciences in universe. Study of differential inclusions under existence of solution and existence of extremal solutions, uniqueness of solutions and various characterizations of solution such as weak solution ,strong solution , asymptotic behavior, stability and attractivity results and its application etc rare work doing in an international level. Some of researchers work on above are as Aoki, M., Astrom, K. J.,Bharucha-Reid, A. T.,K. Deimling, ,Omid S. Fard, Granas and J. Dugundji, Gihman, I. I., and Skorohod, A. V., Izdat. Naukova Dumka, Kiev, S. Heikkila and V. Lakshmikantham, J. Heimmelberg,,S. Itoh, Jazwinsjd, Khasminskii, R. Z.,Kushner, H. J., Ha. Kushner, H. J., Skorohod, A. V.,Tsokos, C. P., and Padgett, W. J.,Wonham, W, M., T.T. Soong

It is clear from the above list of references that differential inclusions have been the subject of rather extensive research. This, however, should not be surprising; for differential inclusions are utilized as models for a wide variety of phenomena considered in the applied mathematical sciences and, consequently, play a dominant role in applied mathematics.

Status:National

Differential inclusions is not detail studied in national level concerned to physical sciences problems but engineering fields are studied by some sense. The study of differential equations and its solution is studied by some researchers are as Srinivasan, S. K., and Vasudevan , R., B.C. Dhage and others. This type of work has been rarely studied at national as well as international level. This work will be a very good contribution in the area of differential equations which is useful for scientist ,engineers, oceanographers and all physical sciences problems.

Year – Wise Plan Of Work

First Year

Survey and Collection of basic materials required for this topic and field work. This includes collection of journals, Books and some research papers related to the topic. Visit for well known institutions, meet and discuss with faculty members for the direction of the topic.

Second Year

The collected material will be processed, analyzed on direction of the research topic and attempt will be made to create the findings in the direction of research topic. The outcome will be represented in national and international gathering such as seminars, workshops. Conferences etc. during this year.

Third Year

The outcome result will be presented in national and international gathering such as seminars, workshops, conferences etc. during this year. Also publishing the outcome results in national, international journals during this year.Writing, and submission of research work to Swami RamanandTeerthMarathwada University ,Nanded.

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